Electrodynamics (PHYS30441) Additional Examples 5

1. In Lecture 15 we worked out expressions for the $\boldsymbol{E}$ field produced by a moving point charge.

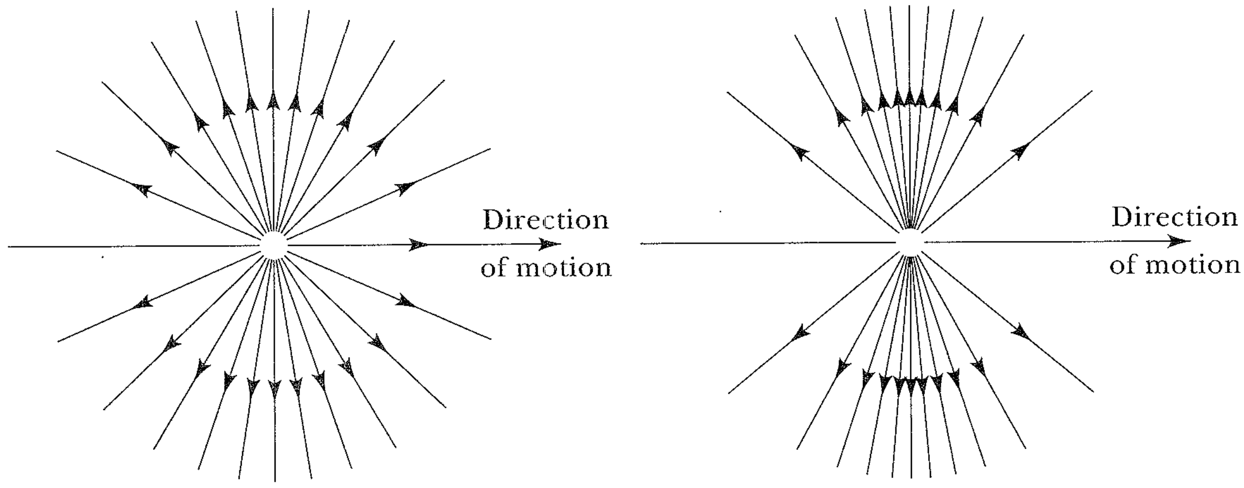


Figure 1: $\boldsymbol{E}$ field of a point charge moving with a constant velocity: $\beta = 0.7$ (left) and $\beta = 0.95$ (right).

   (a) We are going to consider the flux integral $\oint \boldsymbol{E}\cdot d\boldsymbol{a}$ of the $\boldsymbol{E}$ field over a sphere centred at the current position of the moving point charge, $(\beta x^0, 0, 0)$. Can you guess the value of this integral?

   (b) Let's see if your guess is correct! Using the result from Lecture 15 that the magnitude of the total $\boldsymbol{E}$ field can be expressed as

$$E = \frac{q}{4\pi\epsilon_0\gamma^2 R^2}\left(\frac{1}{1-\beta^2\sin^2\theta}\right)^{\frac{3}{2}},$$

   evaluate the flux integral $\oint \boldsymbol{E}\cdot d\boldsymbol{a}$ over a sphere centred at the current position of the point charge $(\beta x^0, 0, 0)$.

   How do you interpret your result?

   You may find the following integral useful:

$$\int \frac{\mathrm{d}u}{(a^2+u^2)^{\frac{3}{2}}} = \frac{u}{a^2\,(a^2+u^2)^{\frac{1}{2}}} + C.$$

2. An electron is released from rest and falls under the influence of gravity in a vacuum. In falling 0.01 m, what fraction of the potential energy lost is radiated away?

3. An uncharged, infinite straight wire carries a current $I(t)$ that is initially zero but rises suddenly to I_0 everywhere in the wire at time $t = 0$. The current may be described by

$$\begin{aligned} I(t) &= 0, \quad \text{for } t \leq 0 \\ &= I_0 \quad \text{for } t > 0. \end{aligned}$$

   (a) Find expressions for the retarded potentials at a point P a distance s from the wire at time t.

   You may wish to use the standard integral:

$$\int \left[a^2+x^2\right]^{-\frac{1}{2}} dx = \ln\left(\left[a^2+x^2\right]^{\frac{1}{2}} + x\right).$$

(b) Hence, show that

$$\boldsymbol{E}(s,t) = -\frac{\mu_0 I_0 c}{2\pi\sqrt{(ct)^2 - s^2}}\,\widehat{\boldsymbol{z}},$$

where $\widehat{\boldsymbol{z}}$ is a unit vector in the direction of the current.

(c) Deduce the magnetic field $\boldsymbol{B}(s,t)$. Comment on the limit of $\boldsymbol{B}(s,t)$ as $t \to \infty$.

(d) Determine an expression for the Poynting vector at point P and time t, and comment on the behaviour at $t = s/c$.

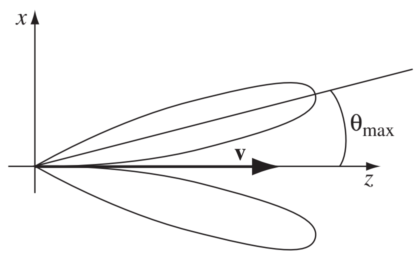
4. As discussed in Lecture 21, the cross check of the total power emitted as Bremsstrahlung radiation requires the evaluation of the integral

$$\int_{-1}^{1} \frac{(1-x^2)}{(1-\beta x)^5}\,\mathrm{d}x = \frac{4\gamma^6}{3},$$

where $x = \cos\theta$. Integrating by parts twice, prove this result.

5. In Lecture 21 we derived an expression for the intensity, $\frac{\mathrm{d}P}{\mathrm{d}\Omega}$, of bremsstrahlung radiation from a point charge.

(a) Find the angle $\theta_{\max}$ at which the maximum radiation is emitted, as shown in the



drawing.

(b) For ultra-relativistic speeds ($\beta = 1 - \epsilon$, where $\epsilon \ll 1$), show that $\theta_{\max} \approx \sqrt{\frac{(1-\beta)}{2}}$.

(c) What is the intensity of the radiation in this maximal direction (in the ultra-relativistic case), in proportion to the maximum intensity for a particle instantaneously at rest? Give your answer in terms of γ.

Note: at the end of term I shall post an additional problem sheet to the course web-page: http://www.hep.man.ac.uk/u/wyatt/electrodynamics.html. This will contain some additional "bonus" problems on various parts of the course to help you with your revision. I shall endeavour to post answers to these bonus problems as soon as possible in the New Year.

Bottom-Line Answers:

1) $(b)\ \dfrac{q}{\epsilon_0}$.

2) Fraction $= \dfrac{\mu_0 ec}{6\pi m}\sqrt{\dfrac{2g}{h}} \approx 2.8 \times 10^{-22}$, where '$m'$ here stands for the rest mass energy of the electron $m = \dfrac{m_{\mathrm{SI}}\, e}{c^2} = 0.511$ MeV.

3) $(a)\ A_z = \dfrac{\mu_0 I_0}{2\pi} \ln K,$ where $K = \dfrac{ct + \left[(ct)^2 - s^2\right]^{1/2}}{s}$;

$(c)\ B_\phi = -\dfrac{\mu_0 I_0}{2\pi}\dfrac{\partial\,[\ln K]}{\partial s} = \dfrac{\mu_0 I_0}{2\pi s}\dfrac{ct}{\left[(ct)^2 - s^2\right]^{1/2}}$;

$(d)\ \boldsymbol{S} = \mu_0 \left(\dfrac{I_0 c}{2\pi}\right)^2 \dfrac{t}{s\left[(ct)^2 - s^2\right]}\,\hat{\boldsymbol{s}}$.

5) $(a)\ \theta_{\max} = \cos^{-1}\left(\dfrac{\sqrt{1+15\beta^2} - 1}{3\beta}\right)$; (c) Ratio $= 2.62\gamma^8$.