Electrodynamics (PHYS30441) Examples 5

1. The LHC at CERN is an example of a synchrotron. The LHC superconducting dipole magnets operate at $\sim 1.9^{\circ}$K and produce a vertical field of around 8 T. The circulating protons have an energy of ~ 7 TeV.

   (a) Using the correct relativistic expression for the momentum of a particle calculate the *bending radius*, R, of the proton beams. This is the radius of the circle in which the particles would travel if they experienced the vertical dipole field around the entire circumference of the accelerator[1].

   (b) Show that the radiated power for a proton within the bending field can be expressed as:
   $$P_{\perp} = \frac{\mu_0 c q^4 B^2 \gamma^2}{6\pi m^2},$$
   and evaluate this quantity for protons in the LHC.

   (c) Hence, in the rest frame of the accelerator, calculate the total energy, $\mathcal{E}$, radiated by a proton in completing one orbit of the LHC. (For this calculation use the circumference of a circle of radius R, rather than the actual circumference of the LHC, which is 27 km).

   (d) Now consider the rest frame of the proton; how would the total energy, $\mathcal{E}'$, radiated by the proton in completing one orbit in the proton rest frame differ from the answer to part (c)?

2. In the LHC the maximum energy to which particles can be accelerated is limited by the strength of the superconducting dipole magnets that bend the paths of particles to follow the (approximately circular) path of the accelerator. During the LHC running period 2015–2018 the maximum beam energy that could be achieved for protons was 6.5 TeV. Lead ($^{208}_{82}$Pb) nuclei are also accelerated in the LHC.

   (a) Using your answer to Q1 part (a) show that the maximum achievable γ for the beam particles in the LHC (given a constant radius and magnetic field strength) is proportional to their charge to mass ratio, Q/M. Hence, show that the maximum energy per nucleon that can be achieved for $^{208}_{82}$Pb nuclei is around 2.5 TeV.

   (b) The radius of a nucleon is around 1×10^{-15} m or 1 fm. Assuming nucleons are densely packed inside the nucleus, estimate the radius of a $^{208}_{82}$Pb nucleus.

   (c) Estimate the electric field strength at the edge of a $^{208}_{82}$Pb nucleus transverse to its direction of motion in the LHC, as measured by an observer at rest with respect to the earth.

   (d) Estimate the electric field strength at the edge of a $^{208}_{82}$Pb nucleus on the axis along its direction of motion in the LHC, as measured by an observer at rest with respect to the earth.

   (e) Comment on the physical origin of the results you have obtained for parts (c) and (d).

Note: at the end of term I shall post an additional problem sheet to the course web-page: http://www.hep.man.ac.uk/u/wyatt/electrodynamics.html. This will contain some additional

[1] The bending radius, R, is is not the actual radius of the LHC ring, since there are other types of magnets (e.g., quadrupoles) in the accelerator as well as eight *straight sections* for the experiments and various other required pieces of the accelerator are located (such as the accelerating RF cavities and the beam dump).

"bonus" problems on various parts of the course to help you with your revision. I shall endeavour to post answers to these bonus problems as soon as possible in the New Year.

Bottom-Line Answers:

1) (*a*) $R = \dfrac{\gamma m v}{qB} \approx 3$ km; (*b*) $P_\perp \approx 10^8$ eV; (*c*) $\mathcal{E} = \dfrac{2\pi R}{c} P_\perp \approx 6.3 \times 10^3$ eV; (*d*) $\mathcal{E}' \approx 0.8$eV.

2) (*b*) 6×10^{-15} m; (*c*) 9×10^{24} V/m; (*d*) 3×10^{21} V/m.