PHYS30441 Electrodynamics: Revision Examples Class

1. (a) Demonstrate that the following vector potential $\mathbf{A}$ is consistent with a homogenous magnetic field $\mathbf{B}$:

$$\mathbf{A} = -\frac{1}{2}\mathbf{r} \times \mathbf{B}.$$

Hint: you may wish to use index notation. Alternatively, you may find the following vector identity useful:
$\nabla \times (\mathbf{v} \times \mathbf{w}) = \mathbf{v}(\nabla \cdot \mathbf{w}) - \mathbf{w}(\nabla \cdot \mathbf{v}) + (\mathbf{w} \cdot \nabla)\mathbf{v} - (\mathbf{v} \cdot \nabla)\mathbf{w}$.

N.B. "homogeneous" means that the vector $\mathbf{B}$ has the same magnitude and direction at all points in space.

(b) Verify that the vector potential given above satisfies the Coulomb gauge. That is, $\nabla \cdot \mathbf{A} = 0$.

2. A flat disk of radius R, carrying a uniform surface charge density σ, is rotating at constant angular velocity ω about an axis that is normal to the surface and passes through its centre. Find its magnetic dipole moment.

3. A rod of length L' lies at rest along the x' axis of a frame of reference S'. Relative to a second inertial frame S, the frame S' moves in the x direction with constant speed β (in units of c). At the time $t = 0$ the rear end of the moving rod passes an observer at rest in S at the origin $x = 0$. *(Hint: you may find it convenient to define this 'event' to occur at $t' = 0, x' = 0$ in frame S'.)* Simultaneously in S the front end of the rod passes a second observer at rest in S at the coordinate x_1.

(a) Using an appropriate Lorentz transformation equation write an expression for x_1 in terms of L'.

(b) Comment on the physical significance of the expression you have obtained above for x_1.

(c) According to an observer in S', at what time t'_1 is the measurement of the front end of the rod made?

(d) Using an appropriate Lorentz transformation equation find the distance, as measured in S', between the points $x = 0$ and $x = x_1$.