Electrodynamics (PHYS30441) Examples 4

1. In lecture 14 we argued that only one point on the world-line of a point-like charged particle can be connected to our "observation point" x^μ (at which we determine the potentials and fields) by a signal travelling at the speed of light.

   Just to make this a bit more concrete, let's consider a point particle travelling at constant speed β (in units of c) along the x^1 axis ($x^2 = 0, x^3 = 0$). Just for convenience we can choose our observation point to be the origin, given by $x^0 = x^1 = x^2 = x^3 = 0$.

   Draw a space-time diagram, showing the world line of the particle and the past light cone of the observation point. Write down a suitable equation for the world line of the particle. Write down also an equation for the past light cone of the observation point. Hence, find the coordinates $(x^0_{\rm ret}, x^1_{\rm ret})$ of the point at which the world line of the particle crosses the past light cone of the observation point.

   [*Hint: your answer will depend on an arbitrary constant that you had to introduce at some point above and also on* β.]

2. A cyclotron is a relatively simple type of particle accelerator, in which the cylindrical accelerator volume is filled with a homogenous, time independent magnetic field, B, that is parallel to the axis of the cylinder. As the particles gain energy (they are accelerated by means of a tangential applied electric field) they spiral outwards from the axis of the cylinder.

   (a) Assume that a particle of charge q and mass m is travelling with non-relativistic velocity in an approximately circular orbit. Show that the frequency with which it orbits is given by
   $$f = \frac{B}{2\pi}\frac{q}{m}.$$
   That is, the orbit frequency depends only on the strength of the applied magnetic field and the charge to mass ratio of the accelerated particle. f is called the "cyclotron frequency". Note that f is independent of the speed of the particle and of the radius of its orbit.

   (b) The particle has (again, non-relativistic) kinetic energy T. Find an expression for the total power radiated as a function of T.
   $$\left[\textit{Answer: } P = \frac{\mu_0 q^4 B^2 T}{3\pi m^3 c}.\right]$$