PHYS30441

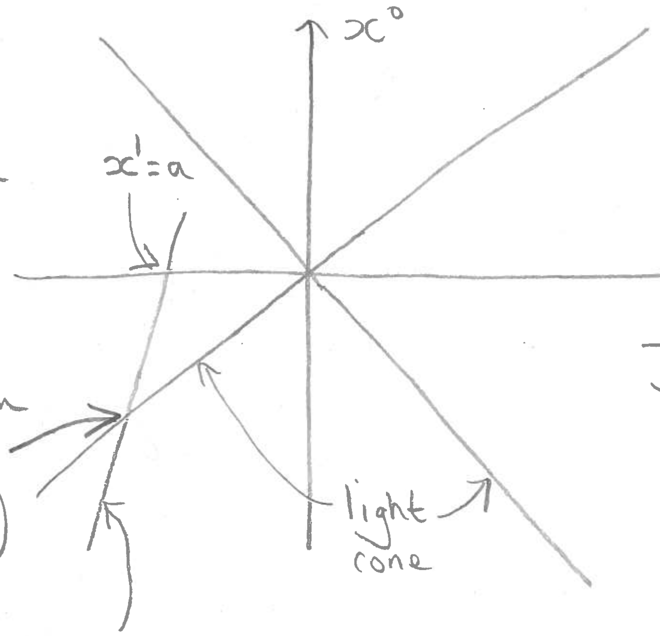
Electrodynamics

Example Sheet 4

Solutions – Terry Wyatt.

Q1)

Space-time point at which particle crosses past light cone of the observation point (x^0_{ret}, x^1_{ret})



Just for convenience let observation point be at $x^0 = x^1 = 0$

World line of particle

$$x^1 = a + \beta x^0 \qquad (\text{as drawn } a < 0 ,\ \beta > 0)$$

From the light cone : $\quad x^0_{ret} = -\left| x^1_{ret} \right|$

From the particle world line : $\quad x^1_{ret} = a + \beta x^0_{ret}$

Combining these equations :

$$x^1_{ret} + \beta \left| x^1_{ret} \right| = a$$

$$a < 0 : \quad x^1_{ret} = \frac{a}{1-\beta}$$

$$a > 0 : \quad x^1_{ret} = \frac{a}{1+\beta}$$

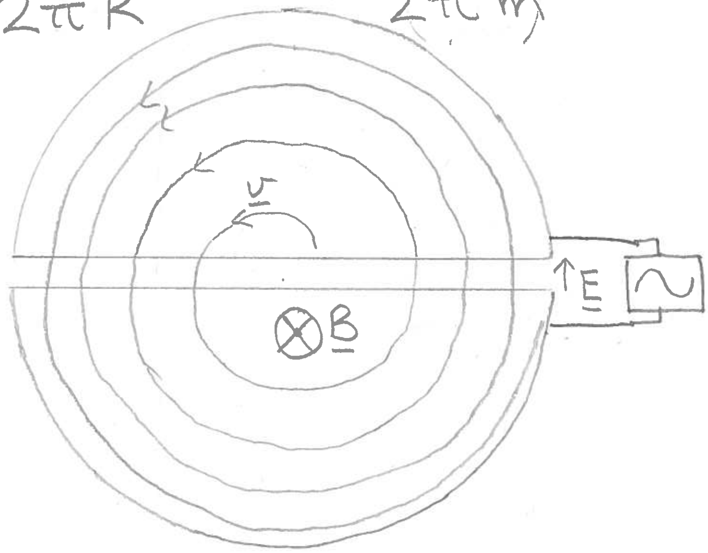
or generally $\quad x^1_{ret} = \dfrac{a}{1 + \beta . \operatorname{sign}(a)}$

$$x^0_{ret} = - \left| \frac{a}{1 + \beta . \operatorname{sign}(a)} \right|$$

Q2) (a) $ma = F = q(\underline{v} \times \underline{B}) = qvB = \frac{mv^2}{R}$

↑ radial component of acceleration

$$f = \frac{v}{2\pi R} = \frac{qB}{2\pi m}$$



Oscillating electric field in gaps between two halves of cyclotron.

(b) $$\therefore \quad a^2 = \frac{q^2 v^2 B^2}{m^2} = \frac{2q^2B^2T}{m^3}$$

$$P = \frac{\mu_0 c q^2 a^2}{6\pi} = \frac{\mu_0 c q^2}{6\pi} \cdot \frac{2q^2B^2T}{c^2 m^3}$$

$$= \frac{\mu_0 q^4 B^2 T}{3\pi c m^3}$$