PHYS30441

Electrodynamics

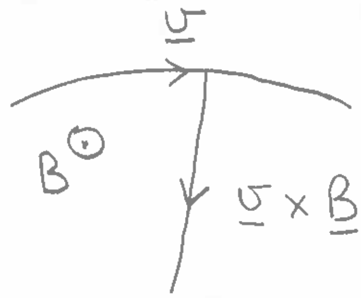
Example Sheet 5

Solutions – Terry Wyatt.

Q1)

(a) For a circular orbit :

$$a = \frac{v^2}{R}$$



In the lectures we showed that the relativisitic expression $\frac{dp^\mu}{d\tau} = qF^{\mu\nu}u_\nu$ reduces to

$$\frac{d\underline{p}}{dt} = q\,\underline{v} \times \underline{B} \quad , \quad \text{since } \gamma\, d\tau = dt$$
$$\text{and } u_\nu = (\gamma c, -\gamma \underline{v})$$

The relativistic expression for momentum is $\underline{p} = \gamma m \underline{v}$

$$\left|\frac{d\underline{p}}{dt}\right| = m\left(v\underbrace{\frac{d\gamma}{dt}}_{\substack{\uparrow \\ \text{which is zero} \\ \text{iff } \underline{a} \perp \underline{v}}} + \gamma \frac{dv}{dt}\right) = \gamma m a = \frac{\gamma m v^2}{R} = qvB$$

$$\therefore \quad R = \frac{\gamma m v}{qB} = \frac{p}{qB} = \overbrace{\frac{7 \times 10^{12} \times e}{c}}^{p = 7\ \text{TeV}/c} \times \frac{1}{8e}$$

$$\boxed{R \approx 3\ \text{km}}$$

N.B. Can write $\quad a = \frac{qvB}{\gamma m} = \frac{qBp}{(\gamma m)^2}$,

which may be useful for part (b)

Q1)

(b) For synchrotron radiation $\underline{a} \perp \underline{v}$

$$P_\perp = \frac{\mu_0 q^2 \gamma^4 a^2}{6\pi c} \qquad \text{since } \dot{\beta}^2 = \left(\frac{a}{c}\right)^2$$

$$= \frac{\mu_0 q^2 \gamma^4}{6\pi c} \times \frac{q^2 B^2}{p^2} \times \frac{p^4}{(\gamma m)^4}, \qquad \text{since } a^2 = \frac{v^4}{R^2}$$

$$= \frac{\mu_0 q^4 B^2}{6\pi c} \frac{(\gamma m c)^2}{m^4} \qquad \text{since } p \approx \gamma m c$$

$$= \frac{\mu_0 c q^4 B^2 \gamma^2}{6\pi m^2}$$

Divide by factor q to get answer in eV!

γ for proton with $E = 7$ TeV, $mc^2 = 0.94$ GeV

$$= \frac{(4\pi \times 10^{-7}) \times (3 \times 10^8) \times (1.6 \times 10^{-19})^3 \times 64 \times \left(\frac{7 \times 10^3}{0.94}\right)^2}{6\pi \times (1.7 \times 10^{-27})^2}$$

$$\approx 10^8 \text{ eV s}^{-1}$$

Q1)

(c) "Circumference" $\approx 2\pi R$

Time for one orbit: $T = \dfrac{2\pi R}{c}$
(in LHC rest frame)

Energy radiated: $\mathcal{E} = \dfrac{2\pi R}{c} P_\perp$

$$\approx 6.3 \times 10^3 \text{ eV}$$

(d) $P_\perp$ is a Lorentz invariant quantity

However, time for one orbit in proton rest frame $T' = \dfrac{T}{\gamma} = \dfrac{2\pi R}{c\gamma}$

Why?

(i) A clock in the proton rest frame measures "proper time"

or, alternatively,

(ii) The circumference of the LHC is Lorentz contracted in the proton rest frame.

$$\therefore \quad \mathcal{E}' = \frac{\mathcal{E}}{\gamma} \approx 0.8 \text{ eV}.$$

Q2) From the answer to question 1

$$\gamma = \frac{q}{m}\frac{RB}{c}$$ for ultra-relativistic particles.

(a) The maximum energy for a given dipole field strength and orbit radius scales as $\left(\frac{\text{charge of the nucleus}}{\text{rest mass}}\right)$ so that for $^{208}_{82}Pb$ we have

$$E \sim 6.5 \times \frac{82}{208} \sim 2.5 \text{ TeV}.$$

(b) Radius of nucleus $\sim \sqrt[3]{\text{number of nucleons}}$

$$\text{radius for Pb: } r \sim \sqrt[3]{208} \times 10^{-15} \text{ m}$$

$$\sim 6 \times 10^{-15} \text{ m}.$$

(c) Perpendicular to direction of motion

$$E \approx \frac{q\gamma}{4\pi\varepsilon_0 r^2} \approx \frac{82 \times 1.6 \times 10^{-19} \times \frac{2.5 \times 10^{12}}{938 \times 10^6}}{4\pi \times 8.85 \times 10^{-12} \times 36 \times 10^{-30}}$$

$$\approx 9 \times 10^{24} \text{ V/m}$$

Q2 (d)

In Lecture 15 we noted that at fixed R'

$E_1 = E_1'$ (equation 15·5)

distance from a point charge in its rest frame.

The point at the edge of the nucleus along the direction of motion corresponds to this case.

$$\therefore \quad E_1 \approx \frac{q}{4\pi\varepsilon_0 r^2} \approx \frac{82 \times 1{\cdot}6 \times 10^{-19}}{4\pi \times 8.85 \times 10^{-12} \times 36 \times 10^{-30}}$$

$$\approx 3 \times 10^{21} \text{ V/m}$$

(e) Because distances $\perp$ to the direction of motion are not Lorentz-contracted, the components of $\underline{E} \perp \underline{\beta}$ always gain a factor of γ when we transform from the rest frame (equation 15·6).

In Lecture 15 we noted that at constant R (distance in the frame S, in which the charge is moving) $E = E'/\gamma^2$ for the component of $\underline{E} \parallel \underline{\beta}$!

However, in this problem we were asked for the field at the edge of the nucleus and the distance from centre to edge is Lorentz-contracted