# A few preliminaries on index notation for vectors in cartesian coordinates

We shall write the "$i$'th" component of a vector $\underline{u}$ in 3-dimensions as:

$$u_i = [\underline{u}]_i \quad , \quad \text{where} \quad i = 1, 2, 3$$

and, in particular, $x_i = [\underline{r}]_i$

Because the three coordinates are orthogonal we have

$$\frac{\partial x_i}{\partial x_j} = \delta_{ij} \begin{cases} = 1 & \text{if } i = j \\ = 0 & \text{if } i \neq j \end{cases}$$

If an index is repeated in an expression it is summed over (unless explicitly stated to the contrary) so that, we can write:

$$\underline{u} = u_i \hat{\underline{x}}_i$$

e.g.

$$\nabla . \underline{u} = \frac{\partial u_j}{\partial x_j}$$

$$\underline{u} \times \underline{v} = \varepsilon_{ijk} \hat{\underline{x}}_i u_j v_k$$

where

$$1 = \varepsilon_{123} = \underbrace{\varepsilon_{312} = \varepsilon_{231}}_{\text{cyclic permutations}} = -\varepsilon_{321} \text{ and other anti-cyclic permutations.}$$

$\therefore \varepsilon_{ijk} = 0$ if any pair of indices are equal.

In the proofs set in the exercises the following relation will be useful

$$\varepsilon_{ijk}\,\varepsilon_{kmn} = \varepsilon_{ijk}\,\varepsilon_{mnk} = \left(\delta_{im}\delta_{jn} - \delta_{in}\delta_{jm}\right)$$