

Formula Sheet

The first three **Legendre Polynomials** are given by

$$P_0(\cos \theta) = 1, \quad P_1(\cos \theta) = \cos \theta, \quad P_2(\cos \theta) = (3 \cos^2 \theta - 1)/2.$$

Vector Calculus

$$\begin{aligned} \nabla(\phi\psi) &= \phi\nabla\psi + \psi\nabla\phi, \\ \nabla(\mathbf{u} \cdot \mathbf{v}) &= \mathbf{u} \times (\nabla \times \mathbf{v}) + \mathbf{v} \times (\nabla \times \mathbf{u}) + (\mathbf{u} \cdot \nabla)\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{u}, \\ \nabla \cdot (\phi\mathbf{u}) &= \phi(\nabla \cdot \mathbf{u}) + \mathbf{u} \cdot (\nabla\phi), \\ \nabla \cdot (\mathbf{u} \times \mathbf{v}) &= \mathbf{v} \cdot (\nabla \times \mathbf{u}) - \mathbf{u} \cdot (\nabla \times \mathbf{v}), \\ \nabla \times (\phi\mathbf{u}) &= \phi(\nabla \times \mathbf{u}) - \mathbf{u} \times (\nabla\phi), \\ \nabla \times (\mathbf{u} \times \mathbf{v}) &= (\mathbf{v} \cdot \nabla)\mathbf{u} - (\mathbf{u} \cdot \nabla)\mathbf{v} + \mathbf{u}(\nabla \cdot \mathbf{v}) - \mathbf{v}(\nabla \cdot \mathbf{u}), \\ \nabla \times (\nabla \times \mathbf{u}) &= \nabla(\nabla \cdot \mathbf{u}) - \nabla^2\mathbf{u}. \end{aligned}$$

Spherical Polar Coordinates

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial r}\hat{\mathbf{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}, \\ \nabla \cdot \mathbf{F} &= \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 F_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta F_\theta) + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}, \\ \nabla \times \mathbf{F} &= \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial \theta}(\sin\theta F_\phi) - \frac{\partial F_\theta}{\partial \phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{\partial}{\partial r}(rF_\phi)\right]\hat{\boldsymbol{\theta}} \\ &\quad + \frac{1}{r}\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial \theta}\right]\hat{\boldsymbol{\phi}}. \end{aligned}$$

Cylindrical Coordinates

$$\begin{aligned} \nabla f &= \frac{\partial f}{\partial s}\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}, \\ \nabla \cdot \mathbf{F} &= \frac{1}{s}\frac{\partial}{\partial s}(sF_s) + \frac{1}{s}\frac{\partial F_\phi}{\partial \phi} + \frac{\partial F_z}{\partial z}, \\ \nabla \times \mathbf{F} &= \left[\frac{1}{s}\frac{\partial F_z}{\partial \phi} - \frac{\partial F_\phi}{\partial z}\right]\hat{\mathbf{s}} + \left[\frac{\partial F_s}{\partial z} - \frac{\partial F_z}{\partial s}\right]\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(sF_\phi) - \frac{\partial F_s}{\partial \phi}\right]\hat{\mathbf{z}}. \end{aligned}$$

Matrix Representations for Relativity

Metric tensor

Lorentz boost along x^1 axis

Electromagnetic field tensor

$$g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad \Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad F^{\mu\nu} = \begin{bmatrix} 0 & -\frac{E_1}{c} & -\frac{E_2}{c} & -\frac{E_3}{c} \\ \frac{E_1}{c} & 0 & -B_3 & B_2 \\ \frac{E_2}{c} & B_3 & 0 & -B_1 \\ \frac{E_3}{c} & -B_2 & B_1 & 0 \end{bmatrix}$$

Radiation from a non-relativistic particle

$$\frac{dP}{d\Omega} = \frac{\mu_0 c q^2}{16\pi^2} \sin^2 \theta \dot{\beta}^2, \quad P = \frac{\mu_0 c q^2 \dot{\beta}^2}{6\pi}.$$