Electrodynamics — Relativity Interactive Session

This question is adapted from a past exam paper

An thin, flat sheet of infinite extent carries a uniform surface charge density σ_0. The sheet lies at rest and extends over the entire x'^1-x'^3 plane of an inertial frame of reference S'.

1. Use Gauss's law to determine the electric field $\boldsymbol{E}$ at a distance x'^2 away from the sheet in frame S'.
2. Write down the electromagnetic field tensor $F'^{\mu\nu}$ corresponding to the fields in frame S'.
3. Consider a second frame of reference S in which the sheet moves with velocity β, in units of c, in the positive x^1 direction.
   (a) State in Lorentz-covariant form the transformation equation that gives the electromagnetic field tensor $F^{\mu\nu}$ in terms of $F'^{\mu\nu}$.
   (b) Use this equation and your answer to part 2. to determine the electromagnetic field tensor $F^{\mu\nu}$ in frame S.
4. Draw a diagram to indicate the velocity of the sheet and the directions of the electric and magnetic fields above and below the sheet in frame S.
5. Give an argument to justify the direction of the magnetic fields based on the electric current, as observed in frame S.
6. Give a physical explanation for the difference in the electric field between the two frames of reference.

   Extensions — if there is time

7. Find a suitable expression for the 4-potential A'^{μ} in frame S'.
8. Apply a Lorentz Transformation to give the 4-potential A^{μ} in frame S.
9. Hence, use the definition of the electromagnetic field tensor

$$F^{\mu\nu} = \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

   to find the non-zero components of $F^{\mu\nu}$.