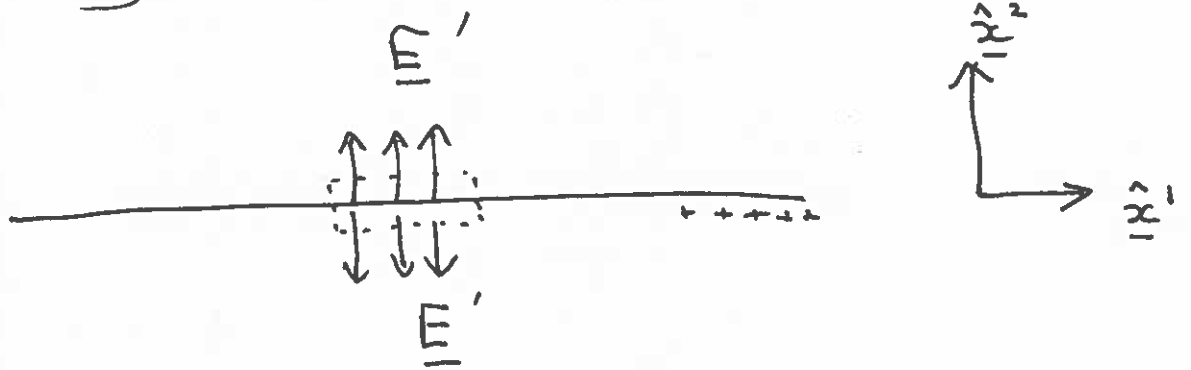
Relativity Interactive Session

1) S'



$$2AE_2' = \frac{A\sigma_0}{\varepsilon_0}$$

For $x^2 > 0$ $\quad E_2' = \frac{\sigma_0}{2\varepsilon_0}$

$x^2 < 0$ $\quad E_2' = -\frac{\sigma}{2\varepsilon_0}$

$$E_2' = \frac{\sigma}{2\varepsilon_0}\,\mathrm{sign}[x^2]$$

By symmetry : $\quad E_1' = E_3' = 0$

$$\underline{B}' = 0$$

Reminder

$$
\Lambda^{\delta}{}_{\gamma} = \begin{array}{c} \\ \delta \downarrow \; 0 \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \gamma \longrightarrow \\ \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\
\left(\begin{array}{cc|cc} \gamma & -\gamma_\beta & & \\ -\gamma_\beta & \gamma & \multicolumn{2}{c}{0} \\ \hline & & 1 & 0 \\ \multicolumn{2}{c|}{0} & 0 & 1 \end{array}\right)
\end{array}
$$

$$
F^{\alpha\beta} = \begin{array}{c} \\ \alpha \downarrow \; 0 \\ 1 \\ 2 \\ 3 \end{array}
\begin{array}{c} \beta \longrightarrow \\ \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\
\left(\begin{array}{cccc} 0 & -E_1/c & -E_2/c & -E_3/c \\ E_1/c & 0 & -B_3 & B_2 \\ E_2/c & B_3 & 0 & -B_1 \\ E_3/c & -B_2 & B_1 & 0 \end{array}\right)
\end{array}
$$

2) $$F'^{\mu\nu} = \begin{bmatrix} 0 & 0 & -E_2'/c & 0 \\ 0 & 0 & 0 & 0 \\ E_2'/c & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3) (a) $$F^{\mu\nu} = \left(\Lambda^{-1}\right)^{\mu}{}_{\alpha} \left(\Lambda^{-1}\right)^{\nu}{}_{\beta} F'^{\alpha\beta}$$

(b) $$F^{\mu\nu} = \underset{\substack{\uparrow \\ \delta^{2}{}_{2}}}{\left(\Lambda^{-1}\right)^{\mu}{}_{2}} \left(\Lambda^{-1}\right)^{\nu}{}_{0} F'^{20}$$

$$\therefore \quad F^{2\nu} = \left(\Lambda^{-1}\right)^{\nu}{}_{0} F'^{20}$$ (only non-zero terms)

Two non-zero terms in $(\Lambda^{-1})^{\nu}{}_{0}$ give:

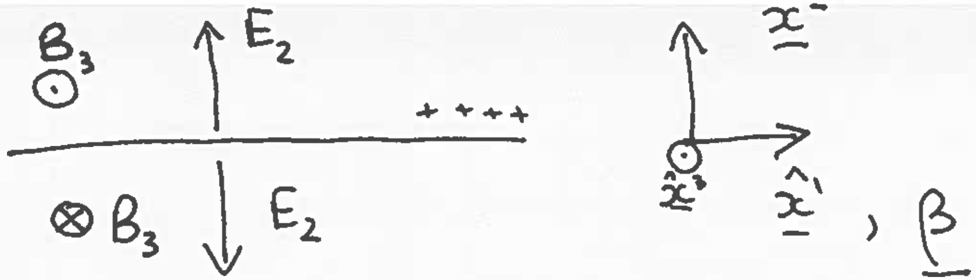
$$\frac{E_2}{c} = F^{20} = (\Lambda^{-1})^{0}{}_{0}\,\frac{E_2'}{c} = \gamma\frac{E_2'}{c}$$

$$B_3 = F^{21} = (\Lambda^{-1})^{1}{}_{0}\,\frac{E_2'}{c} = \gamma\beta\frac{E_2'}{c}$$

$$F^{\mu\nu} = \begin{bmatrix} 0 & 0 & -\gamma\frac{E_2'}{c} & 0 \\ 0 & 0 & -\gamma\beta\frac{E_2'}{c} & 0 \\ \gamma\frac{E_2'}{c} & \gamma\beta\frac{E_2'}{c} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & & \\ 0 & 0 & & \\ E_2/c & \beta E_2/c & & \\ 0 & 0 & & \end{bmatrix}$$

4) S



5) Direction of $\underline{B}$ field make sense from right-hand rule

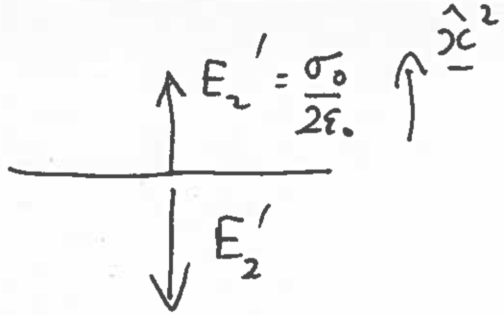
$$\underline{j} \times \underline{r}$$

or from Lecture 15 $\underline{B} = \frac{1}{c} \underline{\beta} \times \underline{E}$

6) Because of length contraction $\Delta x = \frac{\Delta x'}{\gamma}$

$$\sigma_{\text{frame } S} = \gamma \sigma_0$$

7) S'



$$V' = -|x^2| E_2' = -\frac{\sigma_0}{2\varepsilon_0}|x^2|$$

$$A'^{\mu} = \left[-\frac{\sigma_0 |x^2|}{2\varepsilon_0 c}, 0, 0, 0 \right]$$

8) $$A^{\mu} = (\Lambda^{-1})^{\mu}{}_{\nu} A'^{\nu} = [\gamma A'^{0}, \gamma\beta A'^{0}, 0, 0]$$

$$\boxed{= (\Lambda^{-1})^{\mu}{}_{0} A'^{0}}$$

$$= -\frac{\sigma_0 \gamma |x^2|}{2\varepsilon_0 c} [1, \beta, 0, 0]$$

Since A^μ depends only on x2
Need consider only terms in $F^{\mu\nu}$ that contain ∂^2!

9) Only non-zero components in lower (diagonal) half of $F^{\mu\nu}$

Since only A^0, A^1 are non-zero consider only terms in $F^{\mu\nu}$ containing A^0, A^1!

$$\frac{E_2}{c} = F^{20} = \partial^2 A^0 - \underbrace{\partial^0 A^2}_{0} = \frac{\partial A^0}{\partial x_2}$$

$$= \left(-\frac{\partial}{\partial x^2}\right)\left(-\frac{\sigma_0 \gamma |x^2|}{2\varepsilon_0 c}\right)$$

$$= \frac{\gamma \sigma_0}{2\varepsilon_0 c} \frac{x^2}{|x^2|} = \gamma \frac{E_2'}{c}$$

and

$$B_3 = F^{21} = \partial^2 A^1 - \underbrace{\partial^1 A^2}_{0}$$

$$= \gamma\beta \frac{E_2'}{c}$$

as found previously.